Construction and Analysis of Response Surface between Blade Shape and Propeller Characteristics using Multivariate Chebyshev Approximation

Daijiro Arakawa¹, Koichiro Shiraishi¹, Jun Ando²

¹ National Maritime Research Institute (NMRI), Tokyo, Japan
² Faculty of Engineering, Kyushu University (Kyushu U), Fukuoka, Japan

Abstract
Response between Characteristics and shape expressed by design variables exists main effects of a design variable and interaction effects between design variables. Therefore, extracting these effects helps to understand the complex phenomena and design.

In this study, we have developed a new response surface methodology (RSM) and a new proper orthogonal decomposition (POD) using multivariate Chebyshev polynomials. The RSM creates approximate polynomials from the data based on full factor design using orthogonality of the Chebyshev polynomials. The POD makes it possible to visualize the main effects and interaction effects from the regression coefficients of the approximated polynomial. Especially important, these are possible to strictly analyze main effects and complex interaction effects within the sampling data.

In this paper, first we describe outline of the RSM using multivariate Chebyshev polynomials. We construct response surface between propeller blade shape and propeller characteristic, confirm the approximation accuracy. Second, outline of the POD is described. We analyze relationship between propeller blade shape and propeller characteristics. Finally, we visualize main effects and interaction effects of design variables in design space.

Keywords
Marine Propeller, Response Surface Method, Multivariate Chebyshev approximation, Proper orthogonal decomposition, Panel method, Main effect, Interaction effect, Optimization

1 Introduction
With increasing calculation technology applications of optimization method are expanding. Also, proper orthogonal decomposition (POD) is used for wide fields for structural analysis of complex problems. For example, structure, fluid, design, etc (Kosambi 1943) (Brekooz et al 2003) (Bui-Thanh et al 2004).

Recent studies on the marine propeller design using optimization method have conducted (Takekoshi 2005) (Ando 2011). And then, there are also researches using response surface method (RSM) in the optimization process, in order to reduce the calculation cost (Vesting et al 2011). However, approximation accuracy is very important in case using response surface method in the optimization process. In addition, the structure of the characteristic in a design space of the propeller was not extracted. Extracting the main effect and interaction effect of the design variables will greatly help to understand the complex phenomena and design.

In this research, we have developed a new response surface method using multivariable Chebyshev approximation. The response surface method considers the main effect and interaction effect of design variables as much as possible. And we have developed proper orthogonal decomposition using the approximation polynomials. It is possible to observe complex interaction effects using this method.

In this paper, the RSM and the POD using multivariable Chebyshev Polynomials are outlined. We have applied the RSM for marine propeller, and constructed response surface of characteristics of propeller. Moreover, we have extracted main effects of a design variable and interaction effect between design variables, in order to make structure of characteristics of propeller to be simple observable functions or simple imaginable functions.

2 Multivariate Chebyshev approximation
Response surface methodology (Myers 2009) is a method to express a response (objective function) f that depends on the controllable design variables $x_1, x_2, \cdots, x_n$ in an approximation polynomial (response surface) $y$. In this study, multivariate Chebyshev polynomials are applied to

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the approximate polynomial.

The theory of Chebyshev approximation for univariate function is very elegant and known as the best approximation. Additionally, there have been several attempts to extend multivariate function (Rice 1963) (Ryland 2011).

In this section, discrete multivariate Chebyshev approximation is described. Multivariate Chebyshev polynomial used for approximation in this study is expressed by following equation (1). \( a_{m_1, m_2, \ldots, m_n} \) are the unknown regression coefficients. Design variables \( x_1, x_2, \ldots, x_n \) defined by intervals \([a_{i}, \beta_{i}]\) are converted into rationalized variables which become intervals \([-1, 1]\).

\[
y(z_1, \ldots, z_i, \ldots, z_n) = 
\sum_{m_1=0}^{m} \cdots \sum_{m_i=0}^{m} \cdots \sum_{m_n=0}^{m} a_{m_1, m_2, \ldots, m_n} T_{m_1, m_2, \ldots, m_n}(z_1, \ldots, z_i, \ldots, z_n)
\]

(1)

where,

\[
T_{m_1, m_2, \ldots, m_n}(z_1, \ldots, z_i, \ldots, z_n) \text{ are eigen-functions (eigen-vector) as follows.}
\]

\[
T_{m_1, \ldots, m_n}(z_1, \ldots, z_i, \ldots, z_n) = 
T_{m_1}(z_1) \cdots T_{m_i}(z_i) \cdots T_{m_n}(z_n) = \prod_{i=1}^{n} T_{m_i}(z_i)
\]

\[
T_{m}(z_i) = \cos(m_i \cos^{-1} z_i)
\]

The least squares criteria is applied to full factorial design in which each design variable \( x_i \) (i=1,2,\ldots,n) are varied in each levels \( m_{i}+1 \) (i=1, 2, \ldots, n). The regression coefficients can be obtained by stationary condition. This method consists of setting \( m_{i}+1 \) roots of \( T_{m_{i}+1}(z)=0 \) for the levels of full factorial design. Since the orthogonal collocation can be used, we directly compute the regression coefficients \( a_{m_1, m_2, \ldots, m_n} \) by applying equation (3). The regression coefficients \( a_{m_1, m_2, \ldots, m_n} \) are also eigen-values.

\[
a_{m_1, m_2, \ldots, m_n} = 
\frac{\sum_{m_1=0}^{m} \cdots \sum_{m_i=0}^{m} \cdots \sum_{m_n=0}^{m} \int f(x_1, \ldots, x_i, \ldots, x_n)T_{m_1, m_2, \ldots, m_n}(x_1, \ldots, x_i, \ldots, x_n)}{\sum_{m_1=0}^{m} \cdots \sum_{m_i=0}^{m} \cdots \sum_{m_n=0}^{m} T_{m_1, m_2, \ldots, m_n}^2(x_1, \ldots, x_i, \ldots, x_n)}
\]

(3)

Features of the RSM are as follows:

i. The covariance between the different design variables is zero.

ii. The number of sampling data is equal to the number of eigen-values.

iii. Possible to strictly analyze main effects and complex interaction effects with in sampling data.

iv. Possible to directly compute the regression coefficients.

v. Very simple.

3 Response surface of propeller

3.1 Construction of response surface

We create response surfaces for M.P.No.700 which was designed for a coating vessel in National Maritime Research Institute (NMRI) (Kawakita 2017). Principal particular of M.P.No.700 is shown Table 3.1.1.

Pitch and maximum camber distributions in the radial direction are selected as the design variables. Pitch distribution is expressed by Lagrangian interpolation from values at root, 50% radial position (r/R=0.5) and tip. Similarly, maximum camber distribution is expressed from root, 50% and 90% radial position (r/R=0.5 and r/R=0.9). On the whole, there are six design variables in total. Figure 3.1.2 shows the design variables. Table 3.1.2 shows these number i and ranges. And then, the blade section NACA 66 (mod.) a=0.8 is applied. The number of blades, chord, skew, rake and maximum blade thickness distribution in the radial distribution are unchanged.

We construct the response surfaces for thrust coefficient \( K_T \), torque coefficient \( K_Q \), pressure coefficient \( C_p \) using the second-order approximation model with interaction for all design variables. The maximum order is 12 (multiply 6). All in all, it is necessary \( 3^9=729 \) times steady calculations of the propeller performance. We used the simple panel method SQCM (Source and QCM) (Ando 1998) for these calculations. Figure 3.1.1 shows the panel arrangement of M.P.No.700.
We create the response surfaces in two cases. One is in the uniform flow, with the condition of the advance coefficient $J=0.35$. The other is in the circumferential-averaged flow which simulated using wire mesh screen. Figure 3.1.3 shows circumferential-average axial velocity measured in the cavitation tunnel of NMRI. In the wake, the response surfaces were created at the advance velocity where the original propeller matching thrust coefficient in uniform flow $J=0.35$.

### 3.2 Comparison of response surface and SQCM

We confirm the estimated values of the response surfaces for 200,000 propellers that randomly set the design variables within the response surface creation range.

Figure 3.2.1 shows the comparison of $K_T$ and $10K_Q$ between estimated values using the response surfaces and calculated values from SQCM. If red points on the straight line in the figure, it means that the estimated values and the calculated values agree with each other. Summing up, these show that the estimated values of $K_T$ and $10K_Q$ and calculated results are in close agreement.

Figure 3.2.2 shows the comparison of $C_p$ on leading edge panels between estimated values using the response surfaces and calculated values from SQCM. The upper left figure of Figure 3.2.2 shows positions of panels which comparing pressure coefficients. Although the comparison results of $C_p$ show variations compared with the results of $K_T$ and $10K_Q$, the estimated value and the calculated values are in good agreement at the leading edge where the pressure change is drastic.

### 3.3 Optimization

We optimize the propeller blades shape in two way. One uses the response surface in the process of optimization. Another way directly computes propeller performance using SQCM in the process of optimization. The method of giving the objective function and the constraint condition is the same as (Arakawa 2017).

The optimization conditions are in the uniform inflow $J=0.35$ and in the circumferential average flow of the wake simulated by the wire mesh screen.

Figure 3.3.1, 3.3.2 show the pitch distribution and the maximum camber distribution of the prototype propeller and the modified propeller. Table 3.3.1, Figure 3.3.3 show the propeller characteristics. The propeller optimized using the response surface and the propeller optimized using SQCM are 2.3% more efficient than the original propeller (M.P.No.700). The shapes of the two optimized propellers using response surface and SQCM are in very good agreement. Additionally, the time required for evaluation of one propeller of present method (using RSM) is about one hundredth of that of previous method (using SQCM).

Consequently, the results indicate that the response surface method using multivariate Chebyshev approximation has very good approximation accuracy. In other words, these suggest that the effect of design variables.

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Table 3.1.1 Principal particulars of model M.P.No.700

<table>
<thead>
<tr>
<th>Diameter $D_p$ [m]</th>
<th>0.240</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch ratio at 0.7R</td>
<td>0.6059</td>
</tr>
<tr>
<td>Expanded area ratio</td>
<td>0.4791</td>
</tr>
<tr>
<td>Boss ratio</td>
<td>0.1750</td>
</tr>
<tr>
<td>Number of blade $Z$</td>
<td>4</td>
</tr>
<tr>
<td>Skew angle [deg]</td>
<td>25.0</td>
</tr>
<tr>
<td>Rake angle [deg]</td>
<td>4.13</td>
</tr>
<tr>
<td>Blade section</td>
<td>NACA</td>
</tr>
<tr>
<td>Direction of rotation</td>
<td>Right</td>
</tr>
</tbody>
</table>

![Figure 3.1.1 Panel arrangement (M.P.No.700)](image1)

![Figure 3.1.2 Design variables](image2)

Table 3.1.2 Ranges of design variables

<table>
<thead>
<tr>
<th>Design variables</th>
<th>Symbol</th>
<th>$i$</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch ratio at root</td>
<td>$P_1$</td>
<td>1</td>
<td>0.3~1.0</td>
</tr>
<tr>
<td>Pitch ratio at 0.5R</td>
<td>$P_2$</td>
<td>2</td>
<td>0.3~0.9</td>
</tr>
<tr>
<td>Pitch ratio at tip</td>
<td>$P_3$</td>
<td>3</td>
<td>0.3~0.8</td>
</tr>
<tr>
<td>Max. camber ratio at root</td>
<td>$C_{max4}$</td>
<td>4</td>
<td>0.00~0.08</td>
</tr>
<tr>
<td>Max. camber ratio at 0.5R</td>
<td>$C_{max5}$</td>
<td>5</td>
<td>0.00~0.05</td>
</tr>
<tr>
<td>Max. camber ratio at 0.9R</td>
<td>$C_{max6}$</td>
<td>6</td>
<td>-0.005~0.03</td>
</tr>
</tbody>
</table>

![Figure 3.1.3 Circumferential-average axial velocity](image3)
variables can be expressed accurately by the response surface.

4 POD using multivariate Chebyshev polynomial

The proper orthogonal decomposition (POD), also known as Karhunen-Loève transform (KLT) and principal components analysis (PCA), has been widely used for a broad range of application. These are mathematically equivalent. And, these aim to obtain low-dimensional approximate descriptions from high-dimensional problem.

In this section, the POD using multivariate polynomial is described. Approximation polynomial obtained using multivariable Chebyshev approximation is given by a linear combination of the eigen-functions (eigen-vectors) \( T_{k_1, k_2, \ldots, k_n}(z_1, \ldots, z_n) \) as equation (1). In brief, the eigen-vectors mean the structure of the response.

The eigen-value \( a_{k_1, \ldots, k_n} \) depended only on one eigen-vector \( T_{k_1, \ldots, k_n}(z_1, \ldots, z_n) \) tells the contribution of eigen-vector \( T_{k_1, \ldots, k_n}(z_1, \ldots, z_n) \). Put simple, it shows whether the eigen-vectors are stretched or shrunk or reversed or left unchanged. For that reason, we can extract complex effects of design variables to simple functions. Figure 4.1 shows image and example of the POD using multivariable Chebyshev polynomials. It is found that the interaction effect between two different
Example of the POD eigen-values for benchmark functions

Figure 4.1 Image and example of two- dimension POD using multivariable Chebyshev polynomials
variables is greater in Ridge function in (d). Conversely, (e) shows that the variables are influenced independently in Rastrigin function. These results are consistent with expressions of benchmark functions.

In this study, we define contribution of main effect and interaction effect as follow:

i. The contribution of main effect is represented by the eigen-value of the first-order Chebyshev function term of one variable.

ii. The contribution of interaction effect is represented by the eigen-value for the second or higher-order Chebyshev function term or that for multiplication term of Chebyshev function of two or more variables.

Contribution of main effect: \[ a_{\mu_1, \ldots, \mu_n} \left( \sum_{j=1}^{n} \mu_j = 1 \right) \]

Contribution of interaction effect: \[ a_{\mu_1, \ldots, \mu_n} \left( \sum_{j=1}^{n} \mu_j \geq 2 \right) \]

We define total contribution of the interaction effect (TIE) of each design variable using following equation.

\[
TIE_i = \sum_{\mu_1=0}^{m_1-1} \cdots \sum_{\mu_i=0}^{m_i-1} \cdots \sum_{\mu_n=0}^{m_n-1} \left( \sum_{j=1}^{n} \mu_j \right) a_{\mu_1, \ldots, \mu_n} - a_{0, \ldots, 0} \tag{4}
\]

**4.1 Main effect of \( K_T \) and \( K_O \)**

Figure 4.1.1 Contribution of constant and main effect
(Left: \( K_T \), Right: \( 10K_O \))

Figure 4.1.1 shows the contribution of constant and main effect \( K_T \) and \( K_O \) of each design variables in the design space. As result, it analyzes that the contribution of the main effect of \( P_2 \) is the biggest. In contrast, the contribution of the main effect of \( C_{\text{max}4} \) is the smallest. In addition, the contribution of the main effect of \( C_{\text{max}5} \) and \( C_{\text{max}6} \) in the wake are relatively bigger compared with the contribution of these in the uniform inflow.

**4.2 Interaction effect of \( K_T \) and \( K_O \)**

Figure 4.2.1 shows the total contribution of interaction effect \( K_T \) and \( K_O \) of each design variable in the design space. It is shown that the magnitude of that of \( K_O \) is bigger than that of \( K_T \) compared with the main effect. The magnitude of the total contribution of interaction effect for each design variable can be confirmed. The total contribution of interaction effect \( K_T \) and \( K_O \) of \( P_2 \) is the biggest.

There are 729 eigen-values of the approximation polynomial used in this study. The term of the maximum degree is the multiplication of the second-order Chebyshev function of all six variables, it is twelfth-order (2 multiplied by 6 is 12).

Figure 4.2.2, 4.2.3 show the eigen-values of \( K_T \) and \( K_O \) of second-order and third-order terms. The eigen-values of fourth-order to twelfth-order are omitted.

Figure 4.2.2 shows the contributions of interaction effect of \( K_T \). The contribution of interaction effect between \( P_2 \) and \( P_3 \) (\( a_{010100} \)) is the biggest in the uniform inflow condition. While, the contribution of interaction effect between \( P_2 \) and \( C_{\text{max}4} \) (\( a_{001010} \)) is the biggest in the wake inflow condition. Therefore, it can be confirmed that there is a difference in contribution of the interaction effect depending on conditions. Further, some eigen-values of third-order terms that are bigger than these of second-order terms exist. These results confirm there are complex interaction effects in the response surface.

Figure 4.2.3 shows the contributions of interaction effects of \( K_O \). It shows the eigen-values of the multiplication term of \( P_2 \) and \( P_3 \) is big. As a characteristic point, the eigen-values of the multiplication term of \( P_2 \) and \( C_{\text{max}5} \) and that
of P₂ and Cₘₐₓₑ in wake inflow condition are bigger compared with these of uniform inflow condition. It is possible to confirm the difference of effect for design variables between two different conditions.

4.3 Main effect of Cₚ

Figure 4.3.1 shows contours of the eigen-values of the constant term of response surface for pressure coefficient Cₚ. Next, Figure 4.3.2 shows contours of contributions of the main effects of each design variables. The lines and red points below the contours indicate the position of the design variables. We can confirm how each design variable affects the surface pressure distribution, as the main effect.

As for the design variables of the pitch ratio (P₁, P₂, P₃), it is shown the pressure at leading-edge on back-side surface becomes negative, and that on face-side surface becomes positive as the design variable increases. Furthermore, it can be found that the design variable P₂ (Pitch ratio at r/R=0.5) has a strong influence in a wide range.

When the design variable of the maximum camber ratio (Cₘₐₓ₄, Cₘₐₓ₅, Cₘₐₓ₆) increases, the pressure at the leading-edge on back-side surface becomes positive, and that in the adjacent of the mid chord position on back-side surface on back-side surface becomes negative. On the other hand, that at the leading-edge on face-side surface becomes negative, and that in the adjacent of mid chord position on face-side surface becomes positive.

Therefore, the influence of design variables on surface pressure was extracted, using the POD. Additionally, these results are consistent with hydrodynamic interpretation.

4.4 Interaction effect of Cₚ

Figure 4.4.1 shows total contribution of the interaction effect (TIE) of pressure coefficient Cₚ. From this result, the total contribution of interaction effect of Cₚ of P₂ is the biggest. It shows that P₂ widely influences Cₚ on the surface around leading-edge and tip and center of influences the pressure distributions in the adjacent of the
position expressing each parameter. The influence of interaction effects is small compared to the main effects, but it could not be ignored, considering the magnitude of interaction effects.

Representative eigen-values of the second-order terms for $C_p$ response surface are shown in Figure 4.4.2. It means the contribution of interaction effect for $C_p$. That between $P_1$ and $P_2$ at leading-edge near the blade root is negative, while the that of $r/R=0.5$ to the blade tip is positive. That between $P_2$ and $P_3$, and that between $P_2$ and $P_3$ at the leading-edge is negative. Furthermore, that between $P_2$ and $C_{\text{max}5}$, and that between $P_2$ and $C_{\text{max}6}$ at leading-edge are positive, and these also affect negative at the adjacent of the mid chord. The influence of interaction effect between $P_3$ and $C_{\text{max}7}$ is small. It is a natural reason, as the two design variables are physically separate. It is suggested that it is possible to discriminate the magnitude and the position which influence the pressure due to mutual design variables acting.

Figure 4.4.3 shows representative eigen-values of third-order term for $C_p$ response surface. These are smaller than these of second term. But complex interaction effects were visualized at leading-edge.

5 Conclusions
In this study, a new response surface methodology and a new proper orthogonal decomposition using discrete multivariate Chebyshev polynomial has been developed. In these methods can consider and extract as much as possible the interaction effects between design variables from sampling data mathematically. And the theory is very simple, and it is easy to use with simple technology.

The RSM has been applied to propeller characteristics and the blade shape. These results confirm that the propeller
In the future, we aim to apply this method also to the evaluation of propeller cavitation and strength, to obtain information useful for the design. Also, it can be expected to extract useful parameters for the design of the unconventional propellers and energy-saving devices by considering new parameters. For example, shape and stiffness of flexible propellers.

These methods do not give a physical interpretation, but the result is left to the user. However, these can visualize complex interaction effects that could not be visualized so far. It helps to understand how the design variables affect each property. These methods can be applied a wide range of problems.

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