The Coupling of Blade Element Momentum Theory and a Transient Timoshenko Beam Model to Predict Propeller Blade Vibration Response *

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ABSTRACT

The vibration characteristics of a marine propeller is of high interest, especially considering the noise the vibration will produce. To determine the vibration response of a propeller blade caused by a fluid loading, computationally expensive fluid-structure interaction simulations are employed. It is, however, desirable to determine the vibration characteristics using computationally cheap models for use in design tools. In this paper the development of a numerical tool to predict the vibration properties of a marine propeller blade is detailed. The tool uses blade element momentum theory (BEMT) to compute the thrust component along the blade. BEMT is modified to account for the influence of a time dependent wake inflow. The blade structural response is modelled using a 1D transient cantilever beam. The response of the beam was validated using a 3D FEA model of the propeller blade using Ansys, results between the 3D FEA model and the 1D cantilever beam model agree reasonably well in bending, however the beam model is unable to capture natural modes in torsion. It was found that the dominant frequency comes from the rotation rate and the 1st bending mode of the blade. A broad frequency response was found when the flow input is more complex.

Keywords

Fluid-structure interaction, CFD, Vibration, Propellers, Modelling

1 INTRODUCTION

Propellers are commonly used as propulsors for ships and maritime submarines. Unsteady propeller blade motions can have a significant impact on the fatigue life of the propeller and shaft line and increase the noise generation. If the motions excite a vibration mode of the propeller the effect can be significantly detrimental to the fatigue life of the propeller and drive train and be a source of narrow band noise. The motion of the blade also uplifts broadband noise contributions. Excluding cavitation there are two main causes of propeller hydrodynamic noise; swathing and singing. Swathing is the structural response of the propeller blade caused by the unsteady and non-uniform wake inflow. Singing occurs when a trailing edge vortex sheet is shed from the foil and excites one of the vibration modes of the propeller (Sastry 1982). The structural vibration modal frequencies will typically lock-in the vortex frequency rather than that of the shedding frequency for a rigid structure.

The influence of noise uplift is not only a Naval vessel issue but of growing importance because of increasing attention and evidence of its potential impact on the wider marine environment. Typically, a propeller designer mitigates the possibility of blade motion and associated vibrations by designing to reduce blade unsteady forces for a single design condition. This is typically done by careful choice of blade skew and rake distribution. The possibility of blade vibrations is then normally judged based on experience of full scale tests and a limited finite element modal analysis.

Techniques exist to predict the unsteady forces for averaged wake fields. Full transient simulations are not normally done however CFD is being performed to predict ship loads during manoeuvrers and different sea states. Predicting if propeller blade vibration is an issue is a complex task: vibrations can be caused by either or both the unsteady inflow and self-induced vibration from vortex shedding from the trailing edge.

Simulation of this phenomenon using large scale coupled simulations have been developed (Lloyd 2013) ,(Domenico 2018) however, typically these are impractical for design purposes due to their high computational cost. For practical design tools, it is desirable to reduce the computational cost as much as possible. This is done by modelling the fluid loading and structural response using well defined models. In this case the fluid loading is modelled using Blade Element Momentum Theory and the structure of the propeller blade is idealized as a Timoshenko beam.

The aim is to present the simulation tools developed to model the fluid structure interaction response of a propeller blade and eventually of use for design assessment of propeller candidates across a range of operating conditions. These include: (1) the model for a time accurate one-dimensional, six degree of freedom Timoshenko beam (2) implement an unsteady wake into a blade element momentum theory code and obtain transient deformation of

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the propeller blade as it passes through the unsteady wake. This will indicate if the propeller blade natural frequencies are excited by the unsteady wake.

2 METHODOLOGY

2.1 1D Timoshenko Beam Model

To model the structural response of the blade Timoshenko (1921) beam theory was used. Developed in the early 1920s it takes into account shear deformation and rotational bending making it suitable for low aspect ratio beams. This was chosen as it is computationally cheap and can accurately model the structure with relatively few elements. Moreover, the theory has been used to model the structure of horizontal axis wind turbines and helicopter blades (Yardimoglu 2003). Full details of the theory of Timoshenko beams can be found in Andersen et al (2008) .

The Timoshenko model is defined as a series of nodes each with six degrees of freedom with the stiffness matrix of the beam model computed using the moment of inertia, the elastic modulus and the beam geometry. The geometry of the beam is determined by the structural properties of the propeller blade. The length of the beam is the radius of the propeller subtracting the radius of the hub, the thickness of the beam was determined using the maximum thickness of the blade and the breadth determined by the chord at that section. The code to determine the stiffness matrix is capable of changing the chord and thickness of the beam depending on the chord and thickness distribution along the blade. The beam is modelled as a series of cuboidal sections with the height of the element being 70% of the blade thickness and breadth 70% of blade chord to account for the reduced area of the blade section due to the curvature.

This generates a beam capable of motion in 6 degrees of freedom with similar geometric and physical parameters to the propeller blade. To determine the accuracy of the beam model a large scale 3D FEA model was generated using Ansys. The Ansys model used one blade of the PPTC with the hub. The Ansys model consists of 20,000 elements with care to capture the curvature of the blade using increased refinement at the blade tip, trailing edge and leading edge. Figure 1 shows the mesh used in the Ansys model. Three tests were used to determine the accuracy of the beam model in comparison to the Ansys model, these tests were: comparison of static deflection, comparison of transient response, comparison of modal frequencies.

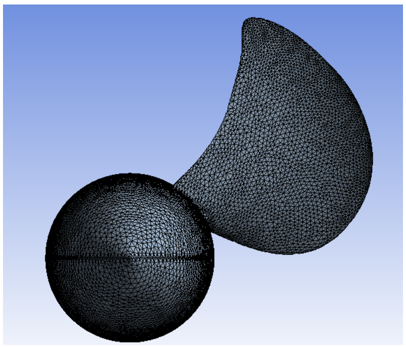


Figure 1: Mesh of PPTC used in Ansys transient analysis

The damping matrix was determined by determining the damping ratio of the 3D FEA model. Firstly, a transient response is developed in the Ansys model by applying a small velocity to the blade tip. A curve fit was added to the transient response of the 3D Ansys model using damping coefficient γ. The general shape of the damping was matched using $y = e^{-\gamma t}$ and γ is adjusted to determine an accurate damping rate. Once an appropriate damping coefficient was determined, the natural frequencies were used to obtain the mass damping coefficient α and the stiffness matrix damping coefficient β as shown in equation 1.

$$\alpha = \frac{\omega_2}{\omega_1}, \quad \beta = \frac{(1-\alpha)\zeta}{\omega_2 - \alpha\omega_1} \tag{1}$$

Where ω_n is the modal frequency of mode n and $\zeta = \frac{\gamma}{\omega_2}$. The damping matrix is then determined using $C = \alpha[M] + \beta[K]$ (Capsoni 2013). Once this damping matrix is determined the transient response of the beam model can be compared to the blade model as shown in figure 2. The modal frequencies are determined through a simple modal analysis where the modal frequencies and shapes are determined by the eigenvalues and eigenvectors. The static analysis is performed by solving the equation $[K]U = [F]$ where [K] is the stiffness matrix U is the deflection vector and F is force matrix

Also, as the propeller will be operating in water it is important to consider the effects of the surrounding fluid on the response of the structure. The Ansys model was placed in a spherical domain of water, with the domain radius only being slightly larger than that of the propeller. The result of placing the propeller in the domain in water is that is lowered the natural frequencies with $\omega_1 dry$ = 1173Hz and $\omega_1 wet$ = 576Hz . The frequency is changed due to added mass. The added mass is modelled in the beam model by assuming each element to be a ellipse at an angle of attack and using equation 2 as found in Bishop (1979).

$$m_{added} = \frac{\rho_{fluid}}{\pi}(t^2\cos^2(\beta_{beam}) + c^2\sin^2(\beta_{beam})) \tag{2}$$

Where ρ_{fluid} is the density of the fluid, t is the thickness of the element, c is the chord and β is the element angle of attack relative to the velocity. Now the stiffness $[K]$, mass $[M]$ and damping matrices $[C]$ have been computed the transient response of the beam can be found. To compute the position of all elements at the next time step the $HHT - \alpha$ method has been implemented.

The Hilber-Hughes-Taylor-α ($HHT - \alpha$) is a generalized method of the $Newark - \beta$ numerical integration method which is an unconditionally stable, implicit scheme. The equation of motion of the beam system is $[M]\ddot{x} + [C]\dot{x} + [K]x = f(t)$. Where x is the displacement of the beam elements and $\dot{x}$ is the time derivative and $f(t)$ is the time dependent forces. Full explanation of the $HHT - \alpha$ method can be found in Hughes (1983). For this case the material properties used was that of stainless steel with Youngs modulus 2.1×10^{11}, Poisson ration ν = 0.3 and density ρ = $7865 kg/m^3$

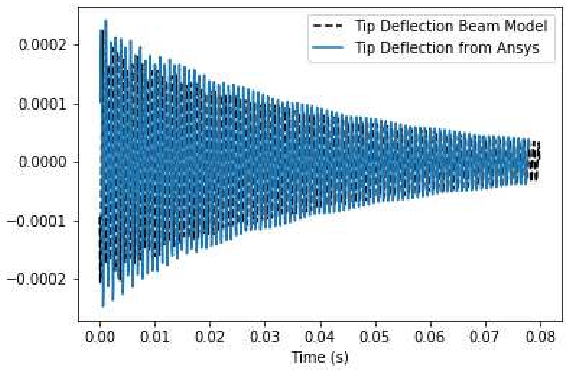


Figure 2: Comparison of tip deflection from beam model and Ansys model

It can be seen that the damping of the blade matches closely to that of the beam, albeit the maximum deflection of the beam model is slightly higher than that of the blade. To further verify the transient response the Fast Fourier Transformation (FFT) of the transient response is presented in figure 3.

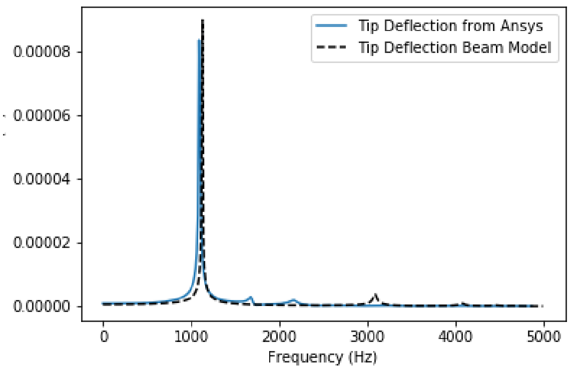


Figure 3: Comparison of the FFT of the Ansys tip deflection and the beam model

It is clear from figure 3 that the vibration response of the beam model and Ansys model match very well in the bending motion with both having a response frequency of $\sim 1100Hz$. However, it can also be seen that the beam model does not capture the torsional response shown at $\sim 1750Hz$. The results of the modal analysis are shown in Table 1 here it is clear that the beam model fails to capture the torsion mode of the Ansys blade model. This is due to the lack of elements in the chordwise direction as when the mode shape of the 2nd mode of the Ansys model is plotted the blade leading and trailing edge are predominantly excited not the bending axis. To model the torsion it would be advisable to replace the beam model with that of a 2D plate. The model is designed to capture the vibration properties of metallic propellers. The use of composite materials will add further complications due to changes in laminate build up which is not modelled using a Timoshenko beam and therefore is not accounted for in this study.

Table 1: Results of Modal analysis.

Ansys Model		
Mode	Frequency (Hz)	Shape
1	1100	Bending
2	1750	Torsion
Beam Model		
Mode	Frequency (Hz)	Shape
1	1100	Bending
2	3100	Bending

2.2 Blade Element Momentum Theory

Blade element momentum theory is useful for modelling propeller performance. This method combines momentum theory, where the propeller is modelled as an infinitely thin annulus with a momentum change, and blade element theory where the propeller blades are modelled as a 2D lifting surface. This method is computationally cheap and has proven to be reasonably accurate. The method is well established and a comprehensive overview can be found in Molland et al (2017) and Burril (1955).

The BEMT algorithm was adapted to allow for the inclusion of a wake inflow. The disc annulus is split into both radial components and circumferential components. This allows for a more accurate description of the force distribution along the annulus whilst also keeping the computational cost low. For this study the disc annulus is split into 12 radial sections to account for the data given for the PPTC and 36 circumferential sections allowing for 10^o per section.

The BEMT code has been verified using previous codes as benchmark cases. Moreover, a CFD case was run to test the open water results and thrust distribution. The details of the CFD are shown on Table 2.

Table 2: Details of CFD simulation

Solver	OpenFoam 5
Method	RANS
Turbulence model	$k-\omega-SST$
Mesh size	~20 million cells
K_T (CFD)	0.60
K_T (BEMT)	0.67

The BEMT and CFD results had good agreement with thrust shown in Table 2. To accurately verify the thrust distribution along each blade the CFD simulation of the PPTC was split into several radial sections to match with the BEMT sections as shown in figure 4.

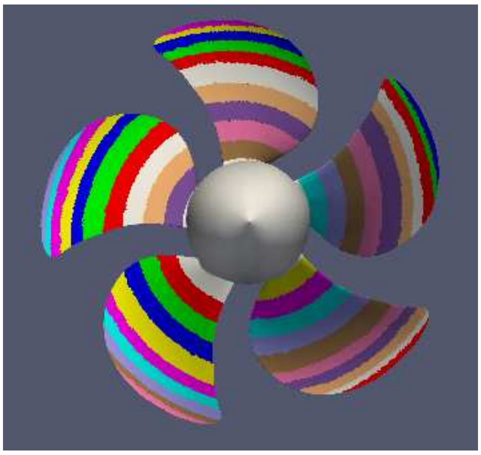


Figure 4: The PPTC split into several radial sections

From figure 4 it can be seen that each radial is not perfectly clean due to the method of computing the blade sections. The blade sections are computed by importing the STL mesh of the propeller and calculating the centre of each triangle. The centres of the triangles are then sorted into radial and angular sections to correspond to the positions set out by the data available in BEMT. The centres are then matched to there corresponding triangles and the STL is produced. This results in small sections of the blade overlapping to the next blade section and must be accounted for

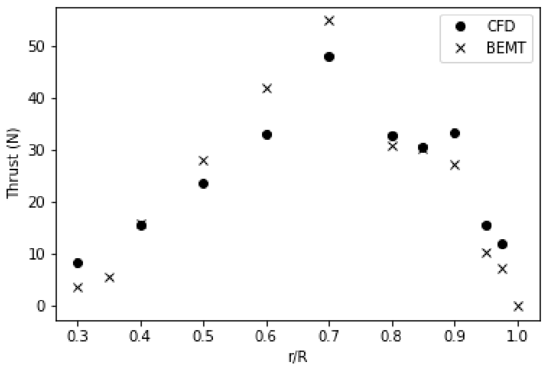
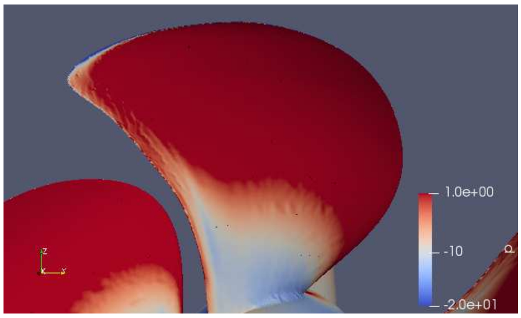
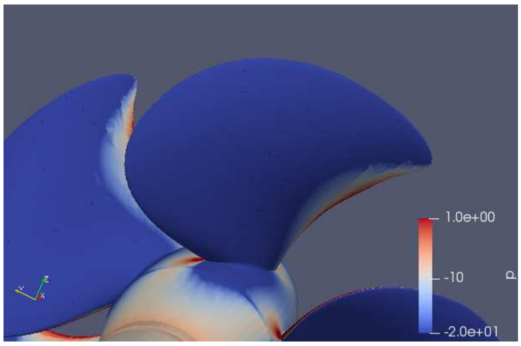


Figure 5: Comparison between radial distribution of thrust per strip of CFD vs BEMT

Figure 5 shows the thrust coefficient against the radial position comparison between the CFD and BEMT thrust results. It can be seen that the thrust from the two methods agree well in parts and are poor in others. The BEMT tends to over-estimate the maximum thrust at position $\frac{r}{R} = 0.7$ whilst slightly underestimating the tip loading. Moreover, BEMT fails to capture the slight uptake in thrust at $\frac{r}{R} = 0.9$ which the CFD captures. The forces are computed in CFD by integrating the pressure on each blade section over the area of that section, therefore the uptake in thrust can be seen when the pressure distribution on the blade is analysed as shown in figure 6a and 6b.



(a) Face side



(b) Back side

Figure 6: Pressure on propeller blade a) face side b) back side

It can be seen from figure 6b that there is a region of lower pressure at the trailing edge of the blade. This region extends from $\frac{r}{R} = 0.4 - 0.8$ with a small increase in pressure at the $\frac{r}{R} = 0.9$ position. This explains the uptake in thrust in this region. Further work would be required to be able to model the thrust distribution along the beam in a more accurate way.

2.3 Coupling Algorithm

To find a time dependent solution to the structural response due to the fluid loading the Timoshenko beam and Blade Element Momentum theory codes must be coupled. This can be done in two ways: i) One-way coupling is where the fluid loading is applied directly to the structure to obtain a response as done in Benra (2011). ii) two-way coupling where the deformation of the structure is iteratively fed back to the fluid solver. The change in geometry will then change the loading on the blades which will then further change the geometry.

2.3.1 One Way Coupling

In this case the one way coupling is achieved by firstly splitting the BEMT output into 36 sections representing 10^o per fluid time step, this keeps computational cost low but limits the frequency range that can be captured. Using the rotational rate it is a simple calculation to determine the loading on the beam at each time step. The loading for the BEMT is appropriately spread over the FEA grid as the FEA grid is much finer than the BEMT grid. The transient response of the beam is solved using a time step of 0.001s and the loading is changed at each timestep as the beam rotates around the wake. To obtain an accurate response a static analysis is performed first to set the initial conditions of the beam bending and deflection. This is to avoid the propeller blade going from unloaded to fully loaded thus creating an impulse force causing the bending frequency of the blade to be excited.

2.3.2 Two Way Coupling

The two way coupling is implemented in a similar way to the one-way coupling. The fluid loading from the BEMT is applied to the beam model. The model is deformed and the the new shape is fed back to the BEMT code and the new loading is found. To update the structure in the BEMT code the twist is added to the pitch distribution of the foil. It has been found in Kiam (2014) that rake has little effect on the thrust distribution of the propeller and is also not accounted for in BEMT, although further improvements could be made to account for this.

Moreover, the hysteresis of the lift coefficient is accounted for using equation 3

$$C_l = C_l xfoil + \frac{\pi c \dot{\alpha}}{2U} \tag{3}$$

Where $C_l xfoil$ is the lift coefficient from xfoil c is the chord, $\dot{\alpha}$ is the velocity of the angle of attack and U is the fluid velocity at that section as described in Hansen (2004).To again avoid an impulse load being applied the loading on the beam is increased slowly. This is done by increasing the rotation rate of the propeller such that as little thrust as possible is produced, the rotation rate is lowered linearly until it reaches 23rps. The algorithm for two-way coupling is:

1. Initialise beam properties.
2. Time step N
3. Run BEMT to get fluid loading
4. Apply loading to Beam model
5. Run Beam model at 0.001s time steps
    (a) Store deformation at each time step
    (b) Update C_l from hysteresis
    (c) Add twist from deformation to pitch for BEMT
6. Repeat from item 2

3 RESULTS

To analyse the performance of the coupling three flows were considered: uniform open water inflow, a generic wake shown in figure 11a and 11b and the wake from a skeg-propeller configuration.

3.1 Uniform flow

The uniform open water case was performed at an advance ratio, $J = 0.6$. The one-way coupling and two-way coupling have been performed. The simulation was run for 3 revolutions of the propeller. The propeller blade studied for these cases is the Potsdam Propeller Test case with chord at 0.75R = 0.106m, thickness at 0.75R = 0.00379m and pitch ratio = 1.635. Stainless steel is used for material properties.

3.1.1 One Way coupling

The maximum deformation occurred at the tip of the blade with a deflection of 0.1mm. Due to the transient nature of the deformation it is difficult to show the deformation in the time domain, therefore the Fast Fourier transformation of the deformation at the tip was performed as shown in figure 7.

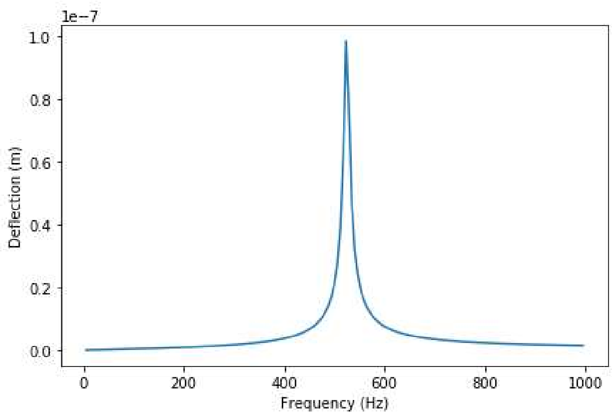


Figure 7: Fast Fourier Transformation of tip deflection of beam from open water case

It can be clearly seen that there is a prominent spike in deflection at the frequency of 528Hz. This corresponds to the 1st natural frequency of the beam. This is expected as the loading on the blade does not change throughout the simulation. It should be noted that the energy at this frequency is very low indicating that there is very little vibration in the open water case.

The deflection of the tip of the beam with time is shown in figure 8. The static deflection overestimates the steady time-dependent value causing a very small oscillation in tip deflection, this oscillation occurs at the first natural frequency of the beam.

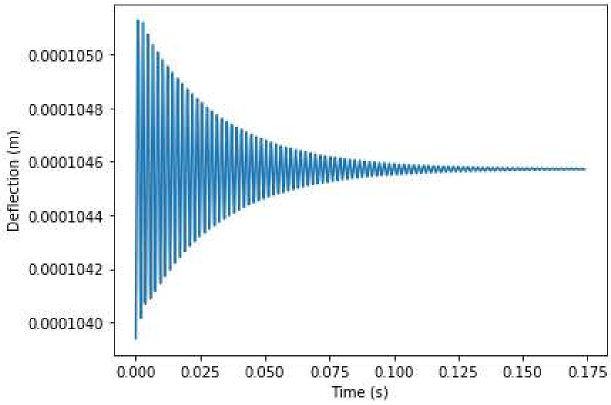


Figure 8: Deflection of Beam tip with time for the One-way coupling of a open water case.

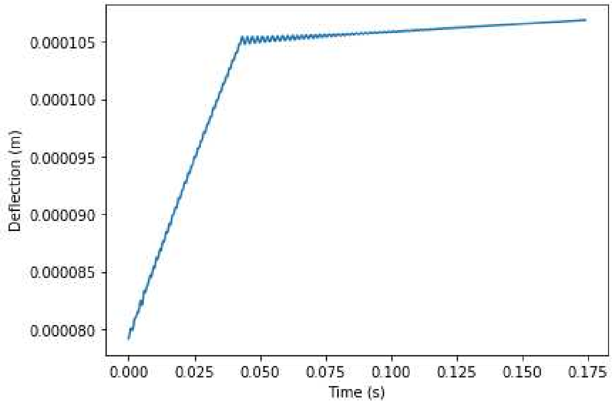


Figure 10: Deflection of Beam tip with time for the One-way coupling of a open water case.

3.1.2 Two-way Coupling

The two-way coupling for the uniform steady flow is then performed with the FFT of the tip deflection shown in figure 9. It is shown that there is very little vibration occurring at this configuration due to the open water inflow. There is however a very slight increase at the first natural frequency.

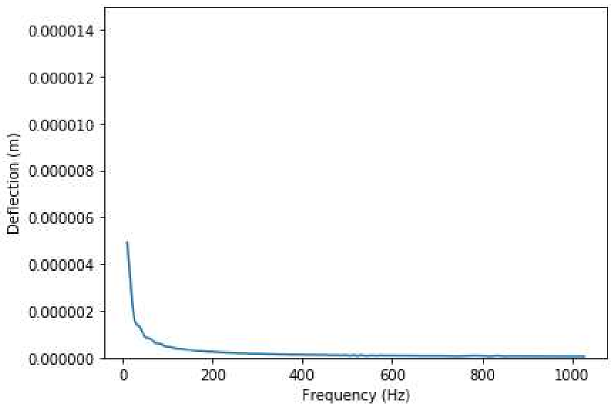
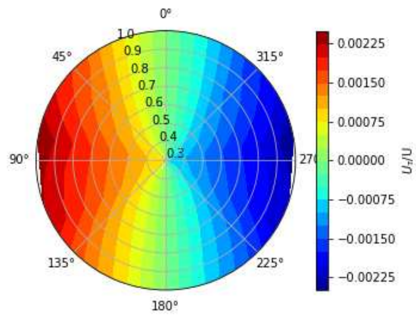


Figure 9: Fast Fourier Transformation of tip deflection of beam from open water case

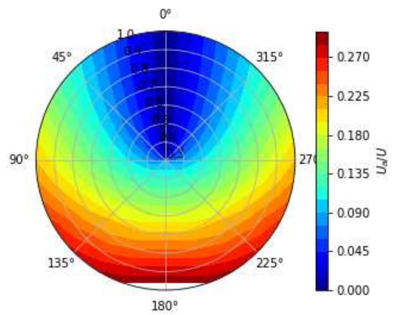
The tip deflection with time can again be seen in figure 10. The deflection can be seen to increase from 0s to 0.04s as the propeller loading is increased until the propeller reaches 23rps where the deflection levels off at approximately 0.0001m. The deflection values of the one-way and two-way are very similar.

3.2 Arbitrary wake

The axial wake input obtained from the arbitrary wake is shown in figure 11 where the inflow to the propeller is outlined is obtained from the data from Hoekstra (1979). This wake represents the slowing of the inlet fluid caused by the presence of the hull with the flow speed increasing as it moves away from the hull. The angular velocity shown in figure 11a is higher on the port side compared to the starboard side, this is attributed to the rotation of the propeller where the blade is on the upstroke the blade is moving against the flow thus causing an increase in velocity experienced by the blade and the opposite is true on the opposing side.



(a) Angular velocity of inflow to BEMT



(b) Axial velocity of inflow to BEMT

Figure 11: Velocity normalised by inlet velocity

3.2.1 One Way coupling

Again, the Fast Fourier Transformation of the signal from the tip deflection of the beam was performed as shown in figure 12. There is several clear spikes in deflection occurring at $N \times 23Hz$ where N is an integer. The highest peak occurs at 23Hz indicating that this is the dominant frequency this is expected due to the cyclic nature of the loading for this generic wake. There is also a small peak at the first natural frequency of the beam indicating there is a small vibration potentially caused by an impulse despite the efforts to load the beam slowly.

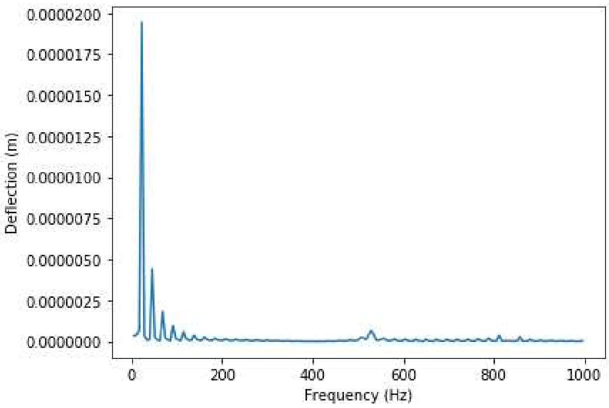


Figure 12: Fast Fourier Transformation of tip deflection of beam from arbitrary wake case.

The cyclic nature of the deflection is clear when shown in figure 13, where the initial deflection is from the static deflection of the beam. The deflection is in a cyclic pattern due to the reduction in thrust at the 6 o'clock position and the increase at the 12 o'clock position. The wake is defined using a cosine function hence the shape of the wake.

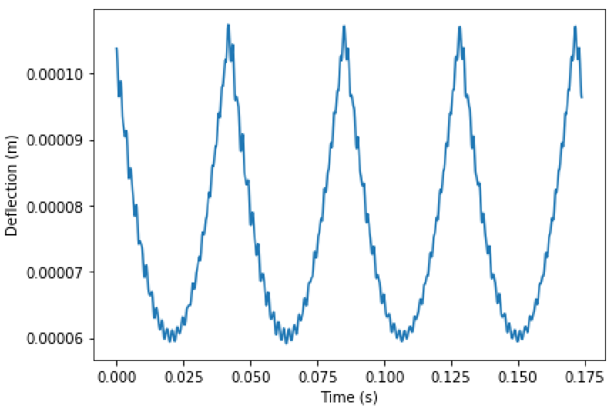


Figure 13: Deflection of Beam tip with time for the One-Way coupling of an arbitrary wake case.

3.2.2 Two-way Coupling

The two-way coupling of an arbitrary wake input is shown in figure 14. The presence of the coupling increases the energy associated to the first bending mode frequency. The increase in energy is caused by the coupling causing the beam to twist causing a greater angle of attack and thus causing increased thrust and therefore greater bending. The slope in the frequency range 0Hz - 200Hz can be attributed to the wind up in thrust as seen in figure 14.

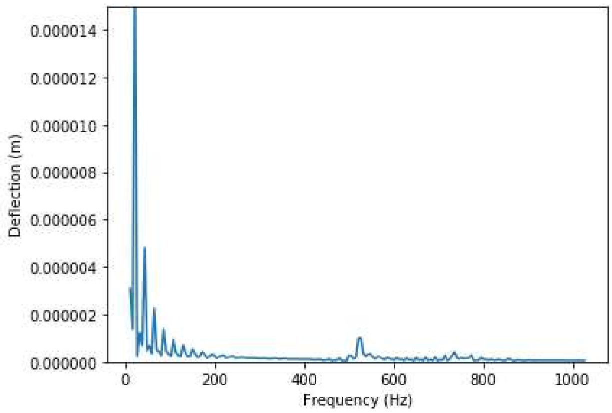


Figure 14: Fast Fourier Transformation of tip deflection of beam from arbitrary wake case.

Figure 14 shows the increase in deflection can be seen between 0 and 0.04s of the model then the cyclic deflection occurs. It is important to note that the cyclic deflection is somewhat arbitrary as the wake data was generated using a cosine-function and is not entirely real for physical cases and the cases discussed are academic to check the new modelling process. It is therefore important to model the response of the beam when subject to a more physical wake.

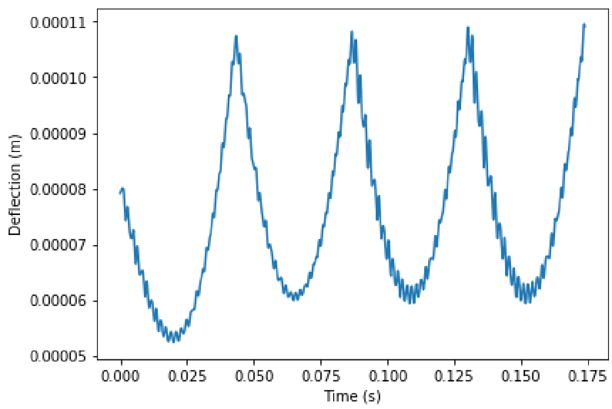


Figure 15: Deflection of Beam tip with time for the Two-Way coupling of an arbitrary wake case.

3.3 Wake from Skeg

To understand the deflection of the blade in a more realistic configuration the inflow to the propeller has been taken as the wake from a skeg-propeller configuration. This is the wake computed by Badoe (2015) as shown in figure 16. Here it can be seen that the flow pattern is far more complex than the inflow in the previous cases therefore a far more complex blade response is expected. Figure 16 shows the axial wake fraction of the inflow as the propeller 'sees' the incoming flow. There is a clear reduction in flow velocity from the 0^o position to the mid point due to the presence of the skeg. There is a region of very low velocity at the mid point to the 180^o position, this is due to the interaction of the fluid with the shaft of the propeller as the skeg is located only on the top half of the propeller shaft.

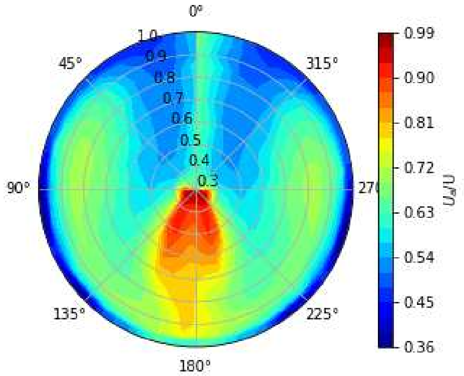


Figure 16: Axial Inflow into propeller from Badoe (2015).

The one and two-way coupling techniques were used as described previously, however the inflow used was that shown in figure 16. The FFT of the response of the blade tip is shown in figure 17. Like the previous responses there is a large peak at the propeller rotation rate and an increase at the 1st bending mode. Moreover, there are far more smaller peaks occurring at a larger range of frequencies when compared to the arbitrary wake case. The cause of these peaks is difficult to determine, however the increase in variable velocity data suggests that the blade will experience a far greater variation in load when compared to previous cases. The tip deflection with time using the one-way deflection is shown in figure 18, the deflection is far noisier compared to previous cases with troughs in deflection occurring every 0.04s and peaks occurring at approximately the same frequency but with a phase lag of 0.015s, these correspond well to the rotation rate of the propeller.

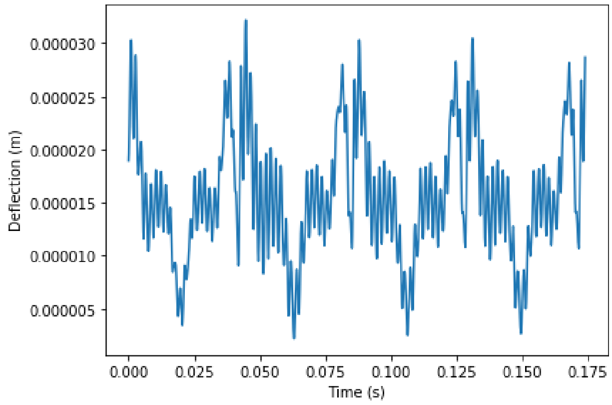


Figure 18: Deflection of Beam tip with time for the one-way coupling of the wake from a skeg.

The two-way coupling is also run with the FFT of the tip deflection shown in figure 19. The two-way coupling broadly keeps the frequency distribution as the one-way coupling however, the higher frequency vibrations tend to be damped out. The overall frequency corresponds well to the one-way coupling albeit the two-way coupling tends to dampen the response. This can be further seen from the tip deflection in figure 20. This has been experienced before in Liaghat (2014) where the fluid-structure interaction of a hydrofoil resulted in much lower deflections when compared to the rigid hydrofoil. This can be attributed to the loading on the blade changing due to the changes in pitch therefore causing a different response.

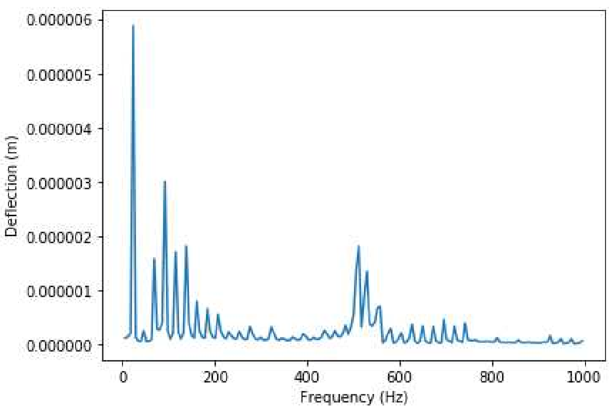


Figure 17: FFT of the response of the beam model to the wake from a skeg using one-way coupling.

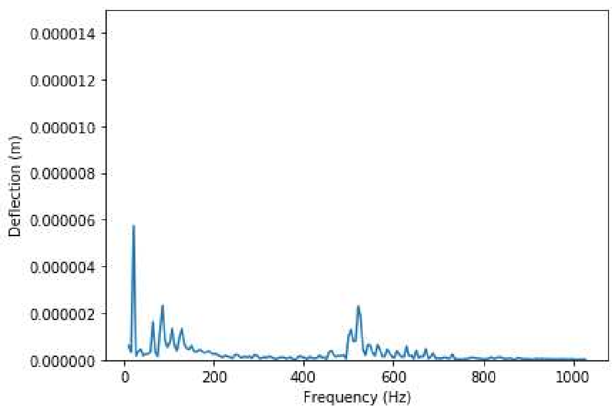


Figure 19: FFT of the response of the beam model to the wake from a skeg using two-way coupling.

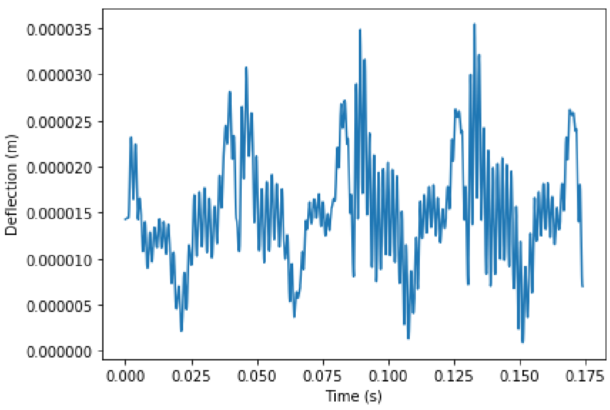


Figure 20: Deflection of Beam tip with time for the Two-Way coupling of the wake from a skeg.

CONCLUSIONS

In this study a design tool has been developed to determine the vibration response of a PPTC blade using blade element momentum theory to determine the fluid loading and a Timoshenko beam model to determine the transient structural response of the blade. The transient, static and modal response of the blade where shown to compare well to a full 3D FEA model of the blade and a CFD case was run to determine the accuracy of the BEMT which gave reasonable results. The tool was run for three wake cases, using both one-way and two-way coupling with the time-domain response of the blade recorded. For all cases the dominant frequencies occurred at the revolution rate of the propeller and the 1st bending mode of the blade. The two sides of the coupling have been shown to work well from larger simulations however validation through experimentation will be required and will be the next step of the study. With validation data some constants and parameters of the models can be adjusted to give a more robust response. Moreover, improvements will need to be made to account for the effects of the blade deformation on the effective wake. Finally as the beam model fails to capture the torsion mode improvement of the structural model will be made. This includes: re-engineering the beam model to account for the location in shear centres or replacing the beam model with that of a plate model. This study is on-going and future work includes comparison between the numerical tool and experimental data, moreover, the propeller geometry being modelled will be changed to give a more robust structural model.

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REFERENCES

Andersen, L., Nielsen, S, 2008, Elastic Beams in Three Dimensions, Aalborg University

Badoe, C. 2015, Design practice for the stern hull of a future twin-skeg ship using a high fidelity numerical approach, Thesis for Doctor of Philosophy, University of Southampton

Benra F K, Hans Josef Dohmen, Ji Pei, Sebastian Schuster, and Bo Wan, 2011, A Comparison of One-Way and Two-Way Coupling Methods for Numerical Analysis of Fluid-Structure Interactions, Journal of Applied Mathematics, vol. 2011, Article ID 853560, 16 pages.

Bishop, R.E.D,1979, Hydorelasticity of Ships, Cambridge University Press

Burril L.C , 1955 , The optimum diameter of marine propellers: A new design approach , Transactions of North East Coast Institution of Engineers and Ship-builders Vol 72 pp 57-82

Capsoni. A, 2013 On damping effects in Timoshenko beams International Journal of Mechanical Sciences, Volume 73, p 27-39

Di Domenico, M. (2018). Fluid structure interaction analysis: vortex shedding induced vibrations. Procedia Structural Integrity

Hansen. MH , Gauna. M Madsen,HA A Beddoes-Leishman type dynamic stall model in state-space and indicial formulations

Hoekstra, M,1977, An investigation into the difference between nominal and effective wakes for two twin-screw ships , Educational and Psychological Measurement

Hughes, T.J.R.,1983, Analysis of Transient Algorithms with Particular Reference to Stability Behavior. in Computational Methods for Transient Analysis North-Holland, pp. 67155.

Kiam Beng Yeo, Rosalam Sabatly, Wen Yee Hau and Cheah Meng Ong, 2014. Effects of Marine Propeller Performance and Parameters Using CFD Method. Journal of Applied Sciences, 14: 3083-3088.

Liaghat, T. Guibault, F. Allenbach, L. Nennemann, B. , 2014 , Two-Way Fluid-Structure Coupling in Vibration and Damping Analysis of an Oscillating Hydrofoil , Volume 4A: Dynamics, Vibration, and Control

Lloyd. (2013). Large eddy simulations of inflow turbulence noise: application to tidal turbines. Thesis University of Southampton.

Molland, A.F., Turnock, S.R., and Hudson D.A., Ship Resistance and Propulsion, Cambridge University Press 2nd Ed(2017)

Sastry, H, 1982, Singing Propellers, Defence Science Journal Volume 32 Issue 1 p 67-73

Timoshenko, S. P., 1921, On the correction factor for shear of the differential equation for transverse vibrations of bars of uniform cross-section, Philosophical Magazine, p. 744.

Yardimoglu, B., 2003, Finite element model for vibration analysis of pre-twisted Timoshenko beam, Journal of Sound and Vibration 273 (2004) 741754