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Calculation of Penetrated Wake Alignments in a Three Dimensional Panel Method for the Rudder Design Process

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ABSTRACT

For the classic design approach of lifting surfaces in naval architecture each device (e.g. propellers, rudders or stabilizers) is treated as a standalone design problem. The panel codes that are often applied to this design purpose use a steady approach and a simplified wake treatment. This procedure is triggered by the project structure of manufacture with a need for short project lead times and furthermore the simplification leads to an improved numerical stability. Especially the calculation of vortex induced velocities in or close to the vortex core leads to singularities in case of standard vortex models. This is not an issue, as long as only one device is considered. Especially to improve the design process of rudders to a higher accuracy it is required to calculate the rudder in the propeller slipstream for typical propeller-rudder arrangements. Another industrial problem, primarily in inland shipping, is the interaction between two rudders in one propeller slipstream, and hence the interaction between two rudder wakes. These problems can be separated into the interaction between a wake and another body and the interaction of two wakes.

This paper discusses a method to calculate the interaction of vortex wakes. The approach will be later used in the above described design purpose. A steady 3D BEM – panel methods including a Kutta - condition for lift generation is utilized for the calculation. The paper concentrates on two important features implemented to improve the behaviour of the wake. The first is the force free wake alignment. This will give later the ability, to take the effect of the (average) non-uniform inflow from the propeller into account. The result is a moveable and subdivided wake, as for unsteady solutions, that is aligned to be force free and consequently having no velocity in the panel normal direction, places itself at the right trailing angle relative to its origin plane. The second is the piercing of two different wake systems. This would lead to numerical instabilities with a classic vortex model. Here a viscous vortex model is implemented in the presented method. Due to this vortex model it is possible to evaluate velocities next to or in the vortex core. This leads to the necessary stability to use the described method in a robust design process of the rudder including the upstream influence of the propeller.

Keywords

3d panel method, wake alignment, penetrated wake panels, rudder design, force free wake alignment, viscous vortex model

1 DESIGN PROCEDURE

The design process of rudders is usually done in iterative steps based on the results done with the previous geometry version. This process is shown in Figure 1.

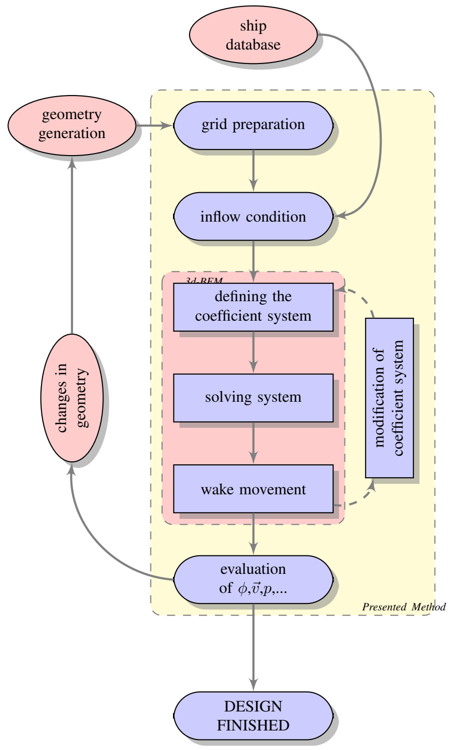

Figure 1: Workflow of the method.

Encircled by the dashed line the calculation method, which is described in this paper, is presented by its major components. To have a smooth and failure robust workflow all the necessary data like restiance curves, loading conditions, hydrodynamic interaction factors and also the rudder geometry is taken from the ships database. The rudder geometry is generated in a fairing module and is in the majority of the time the only factor that is touched in the design process. Due to this, it is especially emphasized in the workflow-chart. Based on the data import from the ship the inflow is determined. The calculation grid is generated directly out of the stored and faired geometry of the rudder.

In the following componets the workflow is quiet similar to the most three dimensional panel methods. First the coefficient system is set up and solved. For the calculation with moveable wakes the velocities in the wake are evaluated and the wake is moved to a new location. The coefficient system is then manipulated based on the wake location and afterwards the system of equations is solved again. This procedure is iterative and aborted after a specific limit is reached. To rate the design, the potential and velocities are evaluated and based on them the forces and cavitational risks are also determined.
At this point the designer has to come to the decision, wether the design could be improved or considered as suitable. Afterwards the force and momentum sets are stored in the database and are used to other calculations, for example manouvering calculations.
Caused by the fact that some design-configurations, like mentioned above, are not able to be calculated, it is necessary to include a votex model that is non-singular at the core into a design tool. This could lead to the benefit, that new design configurations can be tested, which have been discarded before.

1.1 Inflow Condition

For test cases and to have the ability to calculate stabilizer fins or rudders, which are not placed in the propeller slipstream, a uniform inflow is provided. In this case the mean velocity is applied as inflow velocity to all panels.
For the much more common case, where the rudder is placed in the propeller slipstream, the velocities on each panel have to be evaluated. For the design process a lifting line theory is used to calculate the propeller induced velocities in the propeller disc area and afterwards the velocities further downstream are evaluated by an approach according to "Lerbs (1955)". It has to be stated that it would be possible to use the 3d panel-method, which is presented in this paper, for propeller and rudder in interaction. Of course this would gain some difficulties, but the major disadvantage in the design process is the significantly higher computation time. Due to this fact the lifting line theory is used to calculate the inflow to the rudder in the propeller slipstream and to determine the propulsion equilibrium. The upstream effects of the rudder and interaction effects between rudder and propeller wake are neglected due to this approach, but calculation times are kept at a reasonable level.

1.1.1 Lifting Line

To determine the condition in the propeller slipstream it is essential to qualify the propulsional equilibrium precisely. Therefore a lifting line theory is used as described by "Isay (1964)". With this method the forces on the propeller can be predicted quiet well. In the design process only a spatial-continuous result is necessary, therefore a procedure to determine the induced velocities in the propeller disc area based on "Goldstein (1929)" is applied. The calculation is build up as an iterative process, where in the first iteration the induced velocities ($u_q(r) = 0\ ,\ v_q(r) = 0$) are set to zero. The initial solution of the circulation-strength Equation (1) and the hydrodynamic angle of attack Equation (2) is performed. In the next step the Goldsteinfactor is determined based on the ratio between pitch of the unbounded vortices and the normalized radius of the propeller disc.

$$\Gamma(r) \;=\; \frac{\omega\ r\ \tan(\delta_0)\ -\ u_0}{\frac{2}{c_a'}\frac{1}{l\cos(\delta_0)} + \frac{N}{4\pi\kappa}\left(\tan(\delta_0) + \frac{r}{k_0}\right)} \quad (1)$$

$$\tan(\beta_i) \;=\; \frac{k_0}{r} = \frac{u_0\ +\ u_q}{\omega\, r\ +\ v_q} \quad (2)$$

$$(3)$$

The induced velocities in the propeller disc are determined by Equation (4) and Equation (5). And with the results for the induced velocities in the propeller disc area the next iteration is started.

$$u_q(r) \;=\; \frac{r}{k_0}\frac{N\,\Gamma(r)}{4\pi r}\frac{1}{\kappa} \quad (4)$$

$$v_q(r) \;=\; -\frac{N\,\Gamma(r)}{4\pi r}\frac{1}{\kappa} \quad (5)$$

The forces and momentums are derived from the circulation-strength.

1.1.2 Propeller Slipstream Calculation

After the calculation of the induced velocities in the propeller disc area by the Equations (4) and (5), the downstream velocities are performed by Equation (6) and (7). Factor g_a is called Lerbsfactor and taken from "Lerbs (1955)". It becomes one far behind the propeller. This means the value doubles far behind the propeller, what is in good correspondence with pure potential theory. To include a correction for the viscous influence at the boundary of the propeller slipstream a so called 'turbulent mixing correction' according to "Söding (1993)" is done. It is based on the momentum theorem and includes empirical factors.

$$u_q(r,x) \;=\; u_q(r,x=0)\cdot\left(1 + g_a\left(\frac{x}{D},\frac{r}{R_a}\right)\right) \quad (6)$$

$$v_q(r,x) \;=\; v_q(r,x=0)\cdot\left(1 + g_a\left(\frac{x}{D},\frac{r}{R_a}\right)\right) \quad (7)$$

Like also shown by "Greitsch (2006)", a good accordance for induced velocities in the propeller slipstream is delivered by this method.

1.2 Panel Method

The here used three-dimensional steady panel method is based on an approach made by "Söding (1993)" with point doublets and sources. It is developed further with different formulations for the singularity. In detail it is a quadrilateral doublet in the near field and a point doublet in the far field which is based on "Katz (2001)" and as well "Hess and Smith (19xx)". This formulations are used for the on body description of the singularity. The Kutta-condition is treated by the wake. To save calculation time two coefficient matrices are generated. In the first iteration the matrix of all body defined coefficients is generated and stored. Afterwards the coefficients dependend on the wake are generated each iteration and added to the body-matrix to generate the complete system coefficient matrix. This procedure is part of the workflow chart given in Figure 1. The matrix $\mathbf{C}_{wake}$ is only filled in the columns which belong to the trailing edge. Equation (8) illustrates this, with the k$^{\text{th}}$ panel being a trailing edge panel.

$$\mathbf{C} = \mathbf{C}_{body} + \mathbf{C}_{wake} \tag{8}$$

$$= \begin{pmatrix} c_{11} & \cdots & c_{1k_{body}} + c_{1k_{wake}} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots & & \\ c_{j1} & & c_{jk_{body}} + c_{jk_{wake}} & & \vdots \\ \vdots & & \vdots & \ddots & \\ c_{n1} & & & & c_{nn} \end{pmatrix}$$

$$\mathbf{C} \cdot \vec{\mu} = R\vec{H}S \tag{9}$$

$$\mu_k = \mu_{TE_{upper}} - \mu_{TE_{lower}} \tag{10}$$

From the solved system of equations, given by Equation (9), the doublet strength for the wake panles can be found with Equation (10). This is constant over one column of wake panels.

2 WAKE PENETRATION

For cases, when the wake of one rudder is crossing the path of another rudder, the wakes are penetrating each other. Also in the case of a wake roll-up this situation is possible. All situations lead to a singular system of equations and singular results of the potential and velocity evaluation, if a pure potential vortex model is used. "Lamb (1932)" and "Oseen (1912)" have given the first vortex model to include viscosity and ageing of the vortex. A later approach was made by "Vatistas et. al. (1991 and 1998)" and is well verified by experiments. The difference between this models and the potential vortex is the modification of the singularity at the vortex core. The model by "Vatistas et. al. (1991 and 1998)" is applied to the calculation of the influence coefficient and therefore the calculation of the potential and afterwards to the evaluation of the velocities induced by the vortex.

In the following subsection the transformation from a quadrilateral flat panel with constant dipole-strength to a vortex ring is described. This is necessary due to the fact, that the used panel method, like descriebed above, uses dipoles. Furthermore the calculation of the potential, the influence coefficient and the induced velocities are described.

2.1 Transformation of Dipole into a Vortex Ring

Like "Katz (2001)" has shown it is possible to substitute a quadrilateral panel with a constant doublet strength by a closed vortex ring. In the method it is only used for rectengular panels, but it is possible for any closed polygonal. Figure 2 shows the geometrical relations. Here the the point P is the evaluation point, $\vec{r_1}$ and $\vec{r_2}$ are the connection vectors from the end-points of each vortex-segment to P and $\vec{r_0}$ the side-vector. For this specific case the circulation-strength of the vortex ring has to be equivalent to the doublet-strength of the dipole sheet in the panel.

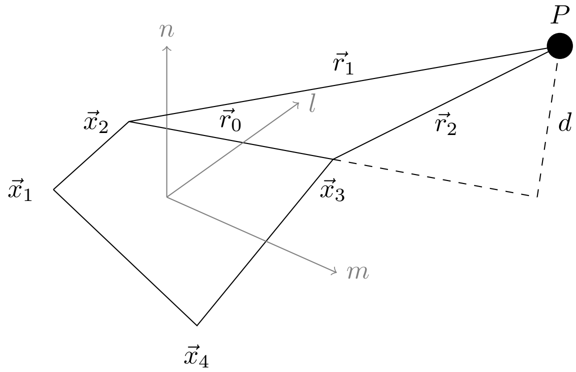

Figure 2: Geometrical relations on the panel.

2.2 Vortex Model

Like stated before a vortex model is needed that does not become singular in the core region. The first vortex-core model was developed by Rankine and is based on the idea of a solid rotating body. This approach is quiet easy, but on the one hand side not physical exact and on the other hand not continuously differentiable. Therefore another classic approach was made by "Lamb (1932)" and "Oseen (1912)", the so called Lamb-Oseen vortex. "Vatistas et. al. (1991 and 1998)" developed and verified another vortex model that has a good accordance with the Lamb-Oseen vortex. The two last named vortex models are both based on the idea that the vortex core region is dominated by viscosity. This effect is not covered by the potential theory. Like mentioned above and shown in Figure 2 the vortex ring is build up by the sum of the edging vortex segments. Hence it is necessary to find a solution for the single vortex segment and afterwards superpose it to the vortex ring. In Equation (11) the solution for one finite potential vortex segment is given, which represents one edge of the vortex ring. Equation (12) gives the numerical implementation.

$$\phi = \frac{\Gamma}{4\pi} \cdot \frac{1}{d} \int_{\beta_1}^{\beta_2} sin(\beta)\, \mathrm{d}\beta \tag{11}$$

$$\phi = \frac{\Gamma}{4\pi} \cdot \frac{1}{d} \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right) \tag{12}$$

$$\frac{1}{d} = \frac{|\vec{r_0}|}{|\vec{r_1} \times \vec{r_2}|} \tag{13}$$

In this formulation the potential is splitted into two terms, first the orthogonal distance to the vortex axis, also given by Equation (13), and second the angular relationship between evaluation point and the endpoints of the vortex segment. For the infinite case ($\beta_1 = 0$ and $\beta_2 = \pi$) the angular relationship term becomes 2 and is so equivalent to the description of the two-dimensional vortex. The formulation for a two-dimensional vortex with a viscous core given by "Vatistas et. al. (1991 and 1998)" is given in Equation (14) with the factor of $n = 2$. Here d_c is the critical distance based on the core radius and d is the acual distance from the evaluation point to the vortex axis, also shown in Figure 2.

$$\phi \;=\; \frac{\Gamma}{2\pi} \cdot \left(\frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \right) \tag{14}$$

This can be transformed into a three-dimensional vortex segment by adding the angular relationship, given by Equation (15). The formulation becomes the original two-dimensional potential vortex, if the associated angular relationship and a core radius equal zero is applied.

$$\phi \;=\frac{\Gamma}{4\pi} \cdot \frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right) \tag{15}$$

In this case the distance term is constructed by the geometric distance, given by Equation (13), and the critical distance, taken from Equation (16). The core radius can be determined (Equation 16) by the kinematic viscosity ν, the Oseen parameter α and the age of the vortex t. For the ageing time it is in the design process acceptable to calculate it from the mean velocity and the distance to the trailing edge $t = |x_{\vec{TE}}|/v_\infty$. To calculate the distance to the trailing edge, the midpoint of the vortex segment is taken as reference point.

$$d_c(t) \;=\; \sqrt{4\alpha\nu t} \tag{16}$$

2.2.1 Influence Coefficient

While the influence coefficient describes the effect of one panel to another under the consideration that the singularity strength is one, the influence coefficient for wake panels itself becomes:

$$c_{jk} \;= \frac{1}{4\pi} \cdot \sum_{k=1}^{N} \left(\frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \right)_k \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right)_k \tag{17}$$

It is necessary to also recalculate the influence coefficient based on the used vortex model, because in the case of a wake penetrition not only the potential and the derived velocities would become singular, even so the system of equations would become singular. Therefore the above description of the coefficient is used.

To calculate the potential of the vortex ring with the applied vortex model the influence of all vortex segements is summed up:

$$\phi_{jk} \;= \frac{\mu_k}{4\pi} \cdot \sum_{k=1}^{N} \left(\frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \right)_k \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right)_k \tag{18}$$

2.2.2 Velocity Evaluation

The procedure to calculate the induced velocities of a vortex ring is similar to the calculation of the potential. First the directional derivative for the potential is formed. This term is independent to the vortex core, because it only includes geometrical information. The formulation of the induced velocities is given by Equation (19).

$$\vec{v} \;=\; \nabla\phi \;= \frac{\Gamma}{4\pi} \cdot \frac{\vec{r_1} \times \vec{r_2}}{|\vec{r_1} \times \vec{r_2}|} \cdot \frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right) \tag{19}$$

In the next step the velocities induced by one vortex segment are superposed to a vortex ring. Like stated before the circulation strength of the vortex segements is equal to the doublet strength of the panel. The velocity induced by one vortex ring panel is given by Equation (20).

$$\vec{v}_{jk} \;= \frac{\mu_k}{4\pi} \cdot \sum_{k=1}^{N} \left(\frac{\vec{r_1} \times \vec{r_2}}{|\vec{r_1} \times \vec{r_2}|} \right)_k \cdot \left(\frac{d}{(d_c^{2n} + d^{2n})^{1/n}} \right)_k \cdot \left(\frac{\vec{r_0} \cdot \vec{r_1}}{|\vec{r_0}| \cdot |\vec{r_1}|} - \frac{\vec{r_0} \cdot \vec{r_2}}{|\vec{r_0}| \cdot |\vec{r_2}|} \right)_k \tag{20}$$

3 WAKE ALIGNEMENT CALCULATION

The location of the wake is in the first iteration cycle set to the half of the rudderangle. This is assumed to be the half of the mean angle of attack. And for a uniform inflow velocity this is the general solution for the free vortex system according to "Betz (1919)". Due to this it is also clear, that the wake surface in the initial solution is an even plane. It is obvious that the induced velocity can not be calculated in the corner points or on the side of a wakepanel independent of the used vortex model. In the situation that a pure potential vortex is used the induced velocities become infinit. For a vortex model with a viscous core the influence by the most dominant vortex becomes zero. This still leads to numerical stable soloutions but the abscence of the dominant vortex produces a non reliable result. Hence the velocity is determined in the panel center and then interpolated with the value of the neighboring panel for the value on the side.

For the tip edges of the wake only the center velocity of the correspondent panel is used. Due to this procedure it is ensured that the dominant vortices are included.

Equation (21) gives the calculation procedure for the new location of the cornerpoint of one wakepanel, represented by $\vec{x}$. Here Δt is a virtual timestep used to adjust the iterative behavior of the calculation.

$$\begin{aligned}\vec{x}_{it=1} &= \vec{x}_{it=0} + ((\vec{v}_{c,left} + \vec{v}_{c,right}) \cdot \frac{1}{2} \\ &+ \vec{v}_{\infty}) \cdot \Delta t\end{aligned} \tag{21}$$

After determining the new location of the wake alignment the coefficient matrix for the wake is recalculated. The system of equations is then set up new and solved again, like described before.

4 RESULTS

Figure 3 shows a calculated wake alignment behind a twisted rudder in the propeller slipstream. It is obvious that the propeller induced velocities in its slipstream dominate the shape of the rudder wake.

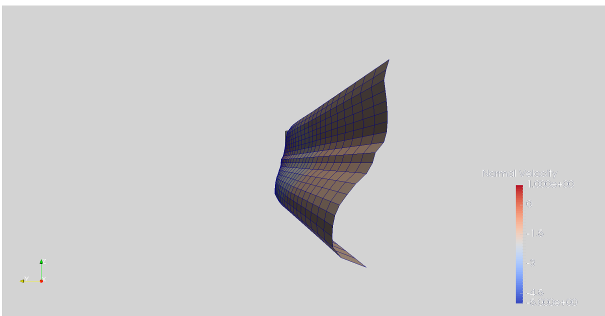

Figure 3: Example of a wake alignment behind a ruddder.

The used vortex model, which is described above, is presented in Figure 4. Also the potential vortex and the Lamb-Oseen vortex is given for comparison. For this figure the core radius is set to one quarter and the circulation-strength is normalized. Here a infinite vortex segment is shown in the orthoganal cutting plane. Due to the fact, that even in one single panel every value for each vortex segment is different, any plot of the superposed velocities would be quiet overloaded.

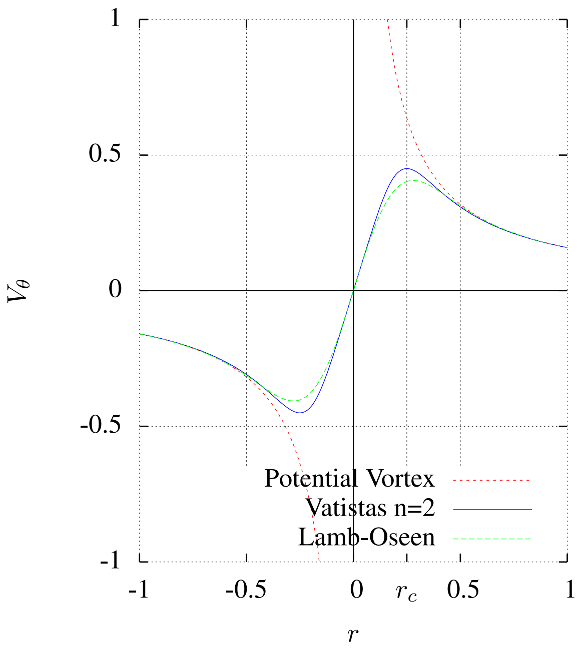

Figure 4: Comparison of different vortex models.

CONCLUSIONS

A procedure to design rudders in unusal arrangements or for new configurations needs a robust and physically correct approach to treat the wake. Especially the case of penetrating wake panels has to be treated, because a random crash of the calculation procedure undermines the confidence not only in the calculation of the wake alignment but also in the method itself. Due to the use of a well verified vortex model with a viscous core and the implementation to the derived factors like the influence coefficients as also the velocities and afterwards the wake location a calculation method for this special design configuration could be generated.

Further development is needed in the question how stretched panels have to be treated in the design process or whether it is tolerable to accept the influence, because it is too small.

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